Six questions about double-diffusive convection

Dan E. Kelley

Dalhousie University, Halifax, Nova Scotia, Canada

Abstract. Double-diffusive convection, at first considered an "oceanographic curiosity," has fascinated fluid dynamicists for a generation. This is partly because some early and basic questions have still not been answered. The most practical of these – whether double diffusion is important in the ocean – was raised in the defining paper, when *Stern* (1960) wrote "*future studies of this model ... will determine whether the proposed mechanism is significant in the vertical mixing of the sea.*" This question of significance has not been answered yet, and may not be until we can answer a host of fundamental questions about mechanisms and interactions, some of which are highlighted here.

Introduction

Double-diffusive phenomena[1] occur in diverse systems, ranging from stars to magma chambers, with oceans in between. Studies of cirrus clouds almost led to the discovery of the mechanism of double diffusion (DD henceforth[2]) in the 1800s (*Schmitt*, 1995), but it was a group of oceanographers who eventually made the discovery a full century later. While pondering whether they could measure deep ocean pressures by lowering pipes from the surface, the oceanographers conceived of a perpetual salt fountain (*Stommel et al.*, 1956), a fanciful idea that soon developed into a theory of SF and of DD convection in general (*Stern*, 1960). An analogy between convecting layers created in the laboratory and layers newly observed in the ocean (*Stommel and Fedorov*, 1967; *Tait and Howe*, 1968) suggested that DD was significant to the ocean, and this was supported by estimates of large oceanic DD fluxes, inferred using laboratory-based flux laws (*Turner*, 1965, 1967). In addition to this work on the case with background T and S fields[3] varying vertically, attention was paid to the case of horizontal variation, as in the theory that divergent DD fluxes might drive interleaving across thermohaline fronts (*Stern*, 1967).

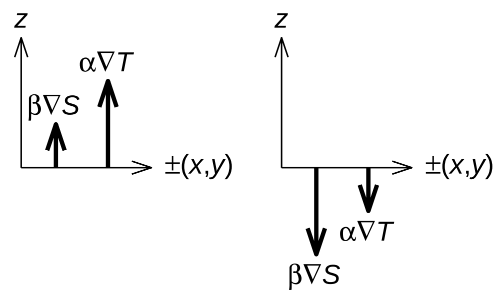

Figure 1. Definition sketch of background gradients in the linear instability view of "staircase" double diffusion (SF in left panel, DC in right panel), with salinity and temperature gradients aligned parallel in the vertical.

The dates in the last paragraph suggest that the key modern ideas about oceanic DD were developed in little more than a decade. However, even that may be an overestimate. *Stommel* (1995) summarized the thought progression from salt fountains to salt fingers under the remarkable heading "Exciting Ten Minutes at the Blackboard." But, whether it took ten minutes or ten years, one thing is clear: the central ideas of DD research came to light in a short time. It might also be noted that these ideas were set out a full generation ago. A student could well ask whether there are significant research questions left for the next generation. I think there are, and I plan to outline some of them here, in hopes of encouraging discussion and future work.

[1] I will assume readers are familiar with double diffusion in the ocean; see recent reviews by *Schmitt* (1994) and *Fernando and Brandt* (1994).

[2] Abbreviations: "DD" for double diffusion; "WS" for relatively warm and salty; "CF" for relatively cold and fresh; "SF" for the salt-finger mode of DD, possible when WS water overlays CF water; and "DC" for the reverse case of diffusive convection.

[3] Notation: T for temperature; S for salinity; α for thermal expansion coefficient; β for haline contraction coefficient; κ for thermal diffusivity; κ_S for haline diffusivity; ν for kinematic viscosity; z for upward coordinate; $R_\rho = (\alpha\partial T/\partial z)/(\beta\partial S/\partial z)$ for the density ratio in the SF case or its reciprocal for the DC case, or either in finite-difference form; Nu for Nusselt number, a nondimensional heat flux; Ra for Rayleigh number, *etc.*, as in *Turner* (1973).

Q1. Where does DD exist in the ocean?

Staircases. Instability theory suggests that DD convection may occur if large-scale gradients of S and T are oriented vertically in the same direction (Figure 1). This sets a very wide domain indeed. For example, *Ingham* (1966) estimated that 90% of the Atlantic Ocean main thermocline has SF-unstable stratification. Furthermore, stratification at high latitudes is very commonly DC-unstable.

A possible test of whether DD is significant in a given region may be whether regular-shaped thermohaline staircases are observed there. This rests on the assumption that regular-shaped staircases can result only if up-gradient DD buoyancy fluxes, which create layers, exceed down-gradient turbulent buoyancy fluxes, which disrupt layers. If we had a solid conception of how DD signatures are formed, it might be feasible to test this notion by comparing spatial patterns of DD signatures with those of mixing rates. Unfortunately, we cannot claim certainty about how DD signatures are formed (see Q2 and Q3 below), nor can we map easily the spatial patterns of mixing rates. Even so, it might be useful to map DD occurrence patterns, in order to guide efforts to understand the DD processes.

For example, it has been noted that SF staircases are mainly seen when R_ρ is less than about 2. Why is this so, and what sets the critical value of R_ρ? Several answers have been put forward. Most of these relate to competition, since instability theory suggests that SF should be possible up to $R_\rho = \kappa/\kappa_S \sim 100$. For example, perhaps the SF growth rate needs to exceed the buoyancy frequency N, if the latter sets a timescale for disruption. Or perhaps it needs to exceed the large-scale shear $\partial U/\partial z$, if SF tilting is the main issue. Or perhaps it needs to exceed the Coriolis parameter f, if inertial turning sets a limit. Other possibilities could be mentioned. While laboratory, theoretical and numerical work on each possibility would be welcome, efforts could be more focused if field studies were undertaken to hint at which physical effects are most relevant. For example, consider the coriolis parameter. *Schmitt* (1994) points out that the *Kunze* (1990) model of disruption by inertial waves implies that the observed maximum R_ρ for regular-shaped SF staircases should decrease with increasing latitude, and that this qualitative pattern seems to be hold in the ocean. Does this provide firm support for the hypothesis of inertial-turning limitation, or is it a coincidence?

The DC case provides a marked contrast to the SF case, since regular-shaped staircases are routinely observed for (DC-formulated) R_ρ values ranging up to at least 10. Is the nature of disruption different in the two cases, *e.g.* with shear inhibiting SF fluxes (*Kunze*, 1994) but not DC fluxes (*Padman*, 1994)? Or is it just that DC survives to higher density ratios because disruption is weaker in regions where DC-unstable stratification exists, such as the Arctic?

Interleaving. Predicting regions in which DD interleaving might occur is more difficult than doing so for staircases. Interleaving has been observed near thermohaline fronts[4] in many regions of the world ocean (*May*, 1999). A dramatic example is provided by the Arctic, which has interleaving signatures with remarkable spatial and temporal coherence (*Rudels et al.*, 1999). To date, sampling of the interleaving mode has been very limited. Extensive field programs are needed to develop a clearer picture of interleaving. It is important that such sampling be on a grid, not on isolated transects, since only then can the along-front and across-front slopes of intrusions be measured. These slopes are key dynamical indicators that might help us to select from competing theories, and select we must. For example, it is still unclear (see Q3 below) whether Arctic interleaving results from DD processes (*May and Kelley*, 2001) or from differential mixing (*Hebert*, 1999; *Merryfield*, 2001), and surely that is a first-order question!

Q2. What creates staircases?

Collective-instability mode. An early hypothesis for staircase formation was a collective-instability mode, in which the SF set up internal waves that in turn disrupt the SF, yielding a system of sheets and layers (*Stern*, 1969; *Stern and Turner*, 1969). Questions remain as to the relevance of the mechanism as a general cause of staircase formation. One is whether the idea can be reformulated for the DC case[5]. Others relate to the details of the proposed mechanism limiting SF length (*Kunze*, 1987, 1990, 1994).

Variable-diffusivity mode. The vertical mode of DD extracts potential energy from the gravitationally destabilizing component of density, transporting buoyancy in the up-gradient direction. Associated flux divergences have been hypothesized as a cause of staircase formation (*Ruddick*, 1997), by analogy to an hypothesis for the creation of steppiness in DD-stable fluids by flux divergences arising from turbulent diffusivities that depend on the buoyancy gradient (*Phillips*, 1972; *Ruddick et al.*, 1989). Preliminary tests along these lines have been carried out via 1D numerical simulations (*Merryfield*, 2000) but questions remain about flux parameterizations, boundary conditions, *etc.* Indeed, the impor-

[4] A thermohaline front is taken here to mean a front with covarying S and T fields, but not necessarily with flat isopycnals. See Figure 2.

[5] However, the search for parallelism between the SF and DC cases owes as much to aesthetic desire as to physical principle.

tance of boundary conditions is difficult to overstate: direct numerical simulations of the SF case show layer formation with insulating top/bottom boundary conditions (*Özgökmen et al.*, 1998) but not with periodic boundary conditions (*Merryfield and Grinder*, 2001).

Shear-modulated mode. The inhibition of SF fluxes by shear (*Linden*, 1971; *Kunze*, 1990, 1994) could lead to vertical variations of buoyancy flux associated with vertical variations in shear, and this has been hypothesized as a mechanism for the generation of fine-structure in SF-unstable regions (*Wells et al.*, 2001). A challenge in taking this to provide a full explanation of staircase formation is the requirement that the spatial pattern of shear match that of observed staircases. Whether that holds or not, this process could play a collaborative role in other mechanisms.

Applied-flux mode. *Turner* (1968a) proposed a mechanism for the creation of thermohaline staircases by the application of destabilizing buoyancy flux, *e.g.* when a salt gradient is heated from below. Further laboratory and theoretical treatments have added details to Turner's initial sketch of this mechanism (*Linden*, 1976; *Huppert and Linden*, 1979; *Fernando*, 1987) and 2D numerical simulations have added color to the picture (*Molemaker and Dijkstra*, 1997). However, questions remain about the directness of the analogy of the applied-flux scenario to the ocean, where staircases appear at mid-depth and where fluxes are likely not to be constant, but rather to depend on the DD response itself.

Modified-intrusion mode. The ideas outlined above are a generation old, but new ideas are now starting to surface. A prime example is the *Merryfield* (2000) proposal that staircases might result from intrusions. The theory produces reasonable predictions of oceanic staircase observations. Issues remaining to be resolved include the role of baroclinicity (*Kuzmina and Rodionov*, 1992; *May and Kelley*, 1997, 2001) and, as usual, the fundamental uncertainty about how to parameterize DD fluxes (see Q5 below). Another issue relates to context: what sets off the initial interleaving? In some cases we may answer that the interleaving results from the contact of watermasses of different TS characteristics. However, in other cases (*e.g.* perhaps in deep Arctic basins) there may not be a great deal of lateral variation in water properties to set off interleaving.

Q3. What creates intrusions?

Neighboring-watermass mode. It is common to observe interleaving across fronts separating WS and CF watermasses. *Stern* (1967) presented an instability theory for this process that has since been extended greatly, *e.g.* allowing for friction as well as diffusion (*Toole and Georgi*, 1981), allowing for more general frontal geometry (*Niino*, 1986), allowing for baroclinicity (*Kuzmina and Rodionov*, 1992; *May and Kelley*, 1997), *etc.* The "allowing for" phrases in the last sentence relate to some of the most basic aspects of ocean physics, and this might suggest that this theory has not borne its last fruit yet. If we think ocean fronts might be unsteady on the timescale of interleaving, if we think cross-frontal contrasts might vary with depth, if we think fronts vary in the downstream direction, if we think diffusivity-based flux laws are flawed, if we think SF and DC fluxes could act at the same time, . . . , then we may not be surprised to see more extensions of the *Stern* (1967) idea. Extending such analytical models of initial growth to the stage of finite-amplitude evolution will remain a challenge. Probably laboratory work will be crucial in guiding thinking, as it has been historically (*Ruddick and Turner*, 1979; *Ruddick et al.*, 1998). Intermediate-scale numerical simulations may play an increasing role in developing understanding, but until we are more certain how to parameterize DD fluxes (see Q5 below) a cloud will hang over such work, as it does now over theoretical treatments.

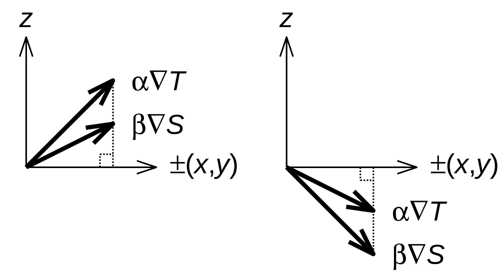

Figure 2. Background gradients in the *Stern* (1967) instability theory of double-diffusive interleaving across a barotropic thermohaline front, for background gradients of the SF (left) and DC senses (right). The dashed lines illustrate that density contributions from S and T are assumed to compensate laterally, yielding flat isopycnals. Is this a good model of ocean fronts?

Sloped-boundary Mode. Sloped insulating boundaries have been shown to create interleaving structures even in fluids with no initial horizontal variations in water properties (*Turner*, 1973; *Linden and Weber*, 1977). This scenario deserves more study, because sloping boundaries are common in the ocean, and thermohaline currents are often steered along them. Might the two mechanisms, the watermass and boundary modes, be linked? A good place to investigate this question might be the Arctic, where interleaving is observed near currents of WS Atlantic waters that appear to be steered along mid-ocean ridges (*Rudels et al.*, 1999).

Differential-mixing mode. Incomplete turbulent mixing can yield differential mixing rates for heat and salt, owing to the difference between the molecular diffusivities of heat and salt acting on fluid parcels momentarily put into contact by the weak turbulence (*Turner*, 1968b; *Altman and Gargett*, 1987; *Ruddick*, 1997). It is not yet clear how to parameterize these mixing rates. The 2D direct numerical simulations of *Merryfield et al.* (1998) confirm the expected, *i.e.* that heat diffuses faster than salt and that the effect vanishes if mixing is vigorous. Soon, we may have flux laws that have been inferred from laboratory work and 3D direct numerical simulations being done today. In the meantime, it is worth noting that the idea of differential mixing may hold promise in answering a long-standing question in intrusion research, namely how to explain intrusions seen in locations that have DD-stable background vertical gradients. The conventional explanations are that (a) the initial lateral displacements were not infinitesimal, as in the theories, but were large enough to create inversions, (b) we are observing the "noses" of intrusions extending from DD-unstable regions into DD-stable regions, or (c) the intrusions are fossilized signatures in a background field that was previously DD-unstable. The idea of differential mixing provides a new hypothesis: that the differential mixing of S and T could yield density convergence analogous to the DD case, thus driving intrusions. This proposal has been put forward recently in a general context by *Hebert* (1999) and for the particular case of Arctic intrusions by *Merryfield* (2001). The former author points out that testing the scenario is problematic in terms of tests in the field, since a key diagnostic is the cross-frontal intrusion slope, which is difficult to measure.

Q4. Do staircases and intrusions interact?

Mixed modes. See the discussion of Q2 for issues relating to the *Merryfield* (2000) idea of transformation of intrusions into staircases.

Intrusions within staircases. Although some staircases display remarkable integrity in some respects (*e.g.* trends in layer TS properties), they are certainly not one-dimensional structures without lateral variation. This is revealed by high-resolution sampling (which is, unfortunately, rare). For example, *Padman and Dillon* (1988) found that station spacings of less than about 1 km were required to track individual layers in the DC staircase that they measured in the Arctic Canada Basin with microstructure temperature profiles. Similarly, towed-chain thermistor sampling of the SF staircase of the C-SALT experiment revealed rich variability on several scales and of several physical types (*Marmorino*, 1989, 1991). For example, within this SF staircase, *Marmorino* (1991) sees evidence of the expected convection plumes within the layers, but also signs of DC interfaces and of intrusions within the layers. These intrusions extend about 1 km laterally, and since they are found in the middle of the wide staircase zone, they seem not to have entered from the edges. What causes them? *Marmorino* (1991) speculates that they may arise because of lateral variations in DD vertical buoyancy fluxes or as a response to mesoscale stirring. These are an important issues to clarify. If lateral phenomena such as intrusions control interface substructures, and if these substructures control DD fluxes, then we won't be able to parameterize DD fluxes in terms of large-scale properties until we can come to grips with lateral affects. Further tests in the field, and in the laboratory, are sorely needed.

Staircases within intrusions. High-resolution sampling sometimes reveals staircases, or at least steppy profiles, between interleaving intrusions (see *e.g. Perkin and Lewis* (1984) Figure 11b). These have received surprisingly little attention to date, probably because sampling has been so sparse. Until fuller understanding is developed, we should view analyses based on presumed DD flux laws between interleaves as being somewhat suspect. The issue may not be easy to resolve, since interleaving environments tend to be more dynamic than staircase environments, and we haven't come to grips with the latter yet.

Q5. Can DD fluxes be parameterized?

Necessity. While diagnostic calculations for a given ocean region can be made using DD fluxes calculated by direct (*e.g.* microscale) measurements, prognostic calculations require a parameterization of DD fluxes in terms of large-scale properties.

Form of flux law. For the DC case it seems likely that layer-layer flux laws (in which fluxes are presumed to be determined by the contrasts ΔS and ΔT between layers) are valid in the ocean (*Padman and Dillon*, 1989; *Padman*, 1994). However, the SF case is apparently much more complicated[6]. It may be that the details of the SF interfaces, as opposed to the layer-layer contrasts, are important in setting fluxes. It may also be that external factors, such as large-scale shear, alter SF convection so much that they must be taken into account in trying to formulate a large-scale diffusivity (*Kunze*, 1994). Since I think we are closer to formulating large-scale parameterizations for the DC case than the SF case, I'll concentrate on the DC case here.

Exponent in layer-layer flux law. Early laboratory work with sharp interfaces between well-mixed

[6] Or has the SF case just been better studied?

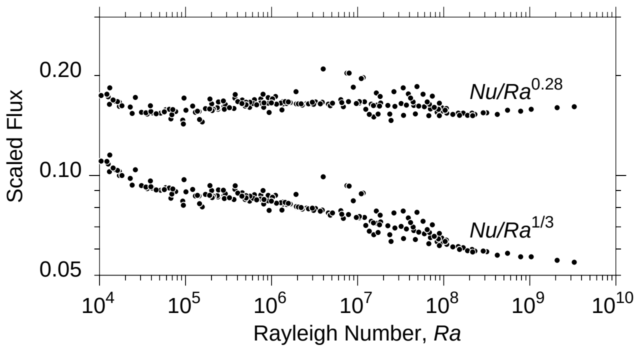

Figure 3. Failure of the 4/3 flux law for thermal convection in water. Dots indicate measurements from five laboratory studies, compiled by *Kelley* (1990). The 4/3 flux law predicts $Nu \propto Ra^{1/3}$ and the lower dots show this is not true; an exponent of 0.28 matches the measurements better.

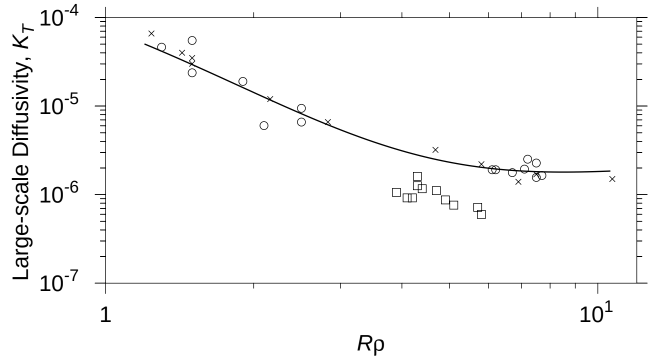

Figure 4. Effective large-scale thermal diffusivity for DC case. The open dots are from *Kelley* (1984), the crosses from *Fedorov* (1988), and the boxes are inferred from the G^* values graphed by *Padman and Dillon* (1987).

layers suggested that vertical fluxes are proportional to $\Delta S^{4/3}$ or $\Delta T^{4/3}$, for the SF or DC cases respectively, with a proportionality factor $C = C(R_\rho)$. This 4/3 exponent was based on a dimensional analysis of single-component convection (*Turner*, 1965). However, measurements of single-component convection contradict the prediction (Figure 3) , yielding an exponent nearer 5/4 than 4/3 (*Kelley*, 1990). A lower exponent is also predicted by convection theories (*Castaing et al.*, 1989; *Kelley*, 1990) and by direct numerical simulations of thermal convection (*Kerr*, 1996). Does the same apply to DD convection? I am unaware of laboratory tests in the DC case, but in the SF case, the laboratory tests are somewhat contradictory. *Schmitt* (1979) reported support for an exponent of 4/3, with regression-based exponents in his Table 3 ranging from 1.24 to 1.37. *McDougall and Taylor* (1984) reported that an exponent of 1.23 matched their observations better than 4/3, but that distinguishing between the two values was problematic with their measurements. Taking these things together, the value of the exponent must be regarded as an open question. One might also ask whether some of the scatter in the empirical value of $C(R_\rho)$ might result from the extrapolation errors resulting from using an incorrect exponent (*Kelley*, 1990).

Linking large-scale properties to vertical fluxes. If fluxes are governed by layer-layer flux laws, as they appear to be in the DC case, then parameterizing fluxes is equivalent to parameterizing the thickness of layers within staircases, since layer thickness together with large-scale gradients yields the ΔS and ΔT values required in order to calculate fluxes. This idea is the gist of a proposed parameterization of large-scale diffusivities for the DC case (*Kelley*, 1984, 1988), illustrated in Figure 4 here. The original measurements are shown in this figure along with the closely matching values of *Fedorov* (1988). However, the values reported by *Padman and Dillon* (1987) are in systematic disagreement, yielding a reduction in the large-scale diffusivity by a factor of approximately 3. The reason for this discrepancy is unknown, and more observations would help to clarify whether the layer-thickness (and diffusivity) scaling presented by *Kelley* (1984) is generally valid. Along similar lines, it would help to examine the pattern of variation of layer thickness, looking for the "split" layers proposed by *Kelley* (1988) to be a signature of a process that controls layer thickness.

Interleaving fluxes. It is not clear how to parameterize interleaving fluxes since the dynamics are still not understood. Even energy-based arguments seem tenuous, given that the energy flow depends on whether the physics involves DD, differential mixing, baroclinic exchange, *etc.* As the C-SALT experiment set a firm foundation for analysis of the SF staircase mode, so might a dedicated field study enliven research on interleaving. In the meantime, gross sensitivity studies in a GCM would be welcome.

Q6. Is DD important?

Locally. In regions with regular-shaped staircase signatures, it seems reasonable to conclude that DD is important compared with other forms of mixing. It has been argued that these are regions with weak turbulent mixing rates, and that this might suggest that DD is not important. However, if a region is of enough interest to foster dynamical study, then the mixing in that region must also be of interest, whether it be large or small. (The Arctic is a prime example.) And what of regions that lack DD signatures? It may be that DD is significant nonetheless. For example, *St.Laurent and Schmitt* (1999) suggest that at the site of the NATRE experiment, where thermohaline staircases were not present, up to half the diffusion of an injected tracer might have been transported by DD.

Globally. Does DD play an important global role, say to the rate of overturning circulation, or to the poleward heat flux, two quantities of great interest for climate studies? One way to tackle such questions is with GCM sensitivity studies. I am unaware of attempts to address the interleaving mode in global domains, either in terms of isopycnal fluxes or diapycnal fluxes (the latter being addressed theoretically by *Garrett* (1982)), but some preliminary studies have been done of the staircase mode. So far, the answer seems to be divided. Some studies suggest a large importance, others a negligible importance. For an example, the GCM simulations by *Zhang et al.* (1998) and *Merryfield et al.* (1999), using similar DD flux parameterizations, yielded very different results. The first study found that including DD mixing reduces the overturning circulation by 22 percent, while the second study found only about 1 percent. Why do these results differ so greatly? Stating differences between the model configurations is straightforward. The former focused on a single basin, the latter on the globe; the former used square walls and zonally-averaged surface forcing, the latter used realistic geography and forcing, *etc.* However, explaining the differences in results is not so straightforward, and may justify further study. Models of intermediate complexity and type might reveal why the *Zhang et al.* (1998) and *Merryfield et al.* (1999) results differ so much. As is usual in ocean models, the form of the surface boundary conditions may be crucial. A followup to the *Zhang et al.* (1998) study, which employed mixed surface boundary conditions instead of the relaxation conditions used by *Zhang et al.* (1998), found that DD had very little affect on the overturning circulation (*Zhang and Schmitt*, 2000). On the other hand, it revealed a heightened sensitivity of the circulation stability to freshwater forcing. Until the contradictory results of such coarse-resolution GCM studies are better understood, the importance of DD fluxes to the global overturning circulation rate remains less than certain.

Answers

Few of the questions listed above were unposed thirty years ago. When will we have answers, and how will we get them? It seems clear that the answers to some of the small-scale questions (*e.g.* how do salt fingers react to shear?) may soon be provided by direct numerical simulations. A new era of oceanographically relevant direct numerical simulations (which I think is imminent; see appendix) may free us from the uncomfortable posture of straddling oversimplified theories and richly-complex laboratory simulations. However, it will be a long time before direct numerical simulations will have the scope to match laboratory (meter) scales, let alone oceanic scales that are orders of magnitude larger. Indeed, if history is any guide, future theoretical and laboratory work, as well as direct numerical simulations, will need field experiments to provide ground truth and also to suggest relevant problems to study. Until we learn how double-diffusive structures are formed in the ocean, and how double diffusion interacts with its competitors, we cannot assess the significance of double diffusion in the vertical mixing of the sea, fulfilling the goal that *Stern* (1960) stated in such sanguine words, so long ago.

Appendix: A computational laboratory?

A 3D numerical simulation on an $N \times N \times N$ grid requires AN^3 numerical operations per timestep, where $A \sim 10^3$ depends on the coding and computer architecture[7]. Thus, a computer that performs F operations per second can do a realtime simulation (directly competitive with a laboratory study) only if $N < (F\Delta t/A)^{1/3}$, where Δt is the model timestep.

Under laboratory conditions, the width of salt fingers is $\sim 3 \times 10^{-3}$m. If $\sqrt{\kappa/\kappa_S} \sim 10$ gridpoints are required for adequate resolution, a reasonable computational mesh might have resolution $\Delta x \sim 3 \times 10^{-4}$m and a diffusive-limit timestep $\Delta t \sim \Delta x^2/\nu \sim 10^{-1}$s. A desktop computer can perform $F \sim 10^8$ operations per second, setting a realtime limit of $N \sim 20$, *i.e.* a domain that can hold under a dozen salt-fingers. This suggests that a desktop computer cannot produce useful realtime simulations. Switching to a supercomputer increases F by a factor of 10^3, so $N \sim 200$ and thus the domain can hold hundreds of salt fingers. Such simulations may be of great utility, but even they span only 1/10th the scale of typical laboratory experiments.

What if the realtime constraint is relaxed? Matching the laboratory domain size by increasing N tenfold increases the computational requirement by 10^3; a year of CPU time would be required to simulate an afternoon in the laboratory. Thus, it seems that the laboratory is the better place for free-wheeling investigation, for exploring parameter space, *etc.*

When will this change? Assuming that computer power continues to double every 1.5 years (*Mann*, 2000), within a decade numerical simulations running at 1/10th realtime (arguably a practical limit) will match laboratory scales. Thus, we may soon enter a new era, in

[7]This estimate of A is perhaps accurate to an order of magnitude, but the accuracy is of little interest here. The main point is the cubic dependence of computational cost on N, which seems inarguable for models based on grids of fixed geometry.

which numerical simulation is a common adjunct to, or a replacement for, laboratory work.

The limiting factor may be software, not hardware. For example, a typical laboratory setup can be described in a few sentences and reproduced both broadly and swiftly, whereas model codes are not easily developed, reproduced, or modified. Formalized schemes for sharing code might prove useful, as they have in the open-source community (*Raymond and Young*, 2001). Funding agencies could help by requiring researchers to use open-source development practices, perhaps sharing codes only after a time delay, as is done with hydrographic data.

Acknowledgments. I thank the *'Aha Huliko'a* organizers for inviting me to this stimulating meeting, as well as the Natural Sciences and Engineering Research Council of Canada and the U.S. Office of Naval Research for funding my work.

References

Altman, D. B., and A. E. Gargett, Differential property transport due to incomplete mixing in a stratified fluid, in *Proceedings of the third international symposium on stratified flows*, Cal. Inst. of Technology, Pasadena, CA, USA, 1987.

Castaing, B., G. Gunaratne, F. Heslot, L. Kadanoff, A. Libchaber, S. Thomae, X.-Z. Wu, S. Zaleski, and G. Zanetti, Scaling of hard thermal turbulence in Rayleigh-Bénard convection, *J. Fluid Mech.*, *204*, 1–30, 1989.

Fedorov, K. N., Layer thickness and effective diffusivities in "diffusive" thermohaline convection in the ocean, in *Small-scale turbulence and mixing in the ocean*, edited by J. C. J. Nihoul and B. M. Jamart, pp. 471–479, Elsevier, Amsterdam, Netherlands, 1988.

Fernando, H. J. S., The formation of a layered structure when a stable salinity gradient is heated from below, *J. Fluid Mech.*, *182*, 525–541, 1987.

Fernando, H. J. S., and A. Brandt, Recent advances in double-diffusive convection, *Appl. Mech. Rev.*, *47*, c1–c7, 1994.

Garrett, C., On the parameterization of diapycnal fluxes due to double-diffusive intrusions, *J. Phys. Oceanogr.*, *12*, 952–959, 1982.

Hebert, D. H., Intrusions: what drives them?, *J. Phys. Oceanogr.*, *29*, 1382–1391, 1999.

Huppert, H. E., and P. F. Linden, On heating a stable salinity gradient from below, *J. Fluid Mech.*, *95*, 431–464, 1979.

Ingham, M. C., The salinity extrema of the world ocean, Ph.D. thesis, Oregon State Univ., Corvallis, OR, USA, 1966.

Kelley, D. E., Effective diffusivities in ocean thermohaline staircases, *J. Geophys. Res.*, *89*, 10, 484–10, 488, 1984.

Kelley, D. E., Explaining effective diffusivities within diffusive oceanic staircases, in *Small-scale turbulence and mixing in the ocean*, edited by J. C. J. Nihoul and B. M. Jamart, pp. 481–502, Elsevier, Amsterdam, Netherlands, 1988.

Kelley, D. E., Fluxes through diffusive staircases: a new formulation, *J. Geophys. Res.*, *95*, 3365–3371, 1990.

Kerr, R. M., 1996, Rayleigh number scaling in numerical convection, *J. Fluid Mech.*, *310*, 139–179.

Kunze, E., Limits on growing, finite-length salt fingers: a Richardson number constraint, *J. Mar. Res.*, *45*, 533–556, 1987.

Kunze, E., The evolution of salt fingers in inertial wave shear, *J. Mar. Res.*, *48*, 471–504, 1990.

Kunze, E., A proposed flux constraint for salt fingers in shear, *J. Mar. Res.*, *52*, 999–1016, 1994.

Kuzmina, N. P., and V. B. Rodionov, Influence of baroclinicity on formation of thermohaline intrusions in ocean frontal zones, *Izvestiya, Atm. & Oce. Phys.*, *28*, 804–810, 1992.

Linden, P. F., Salt fingers in the presence of grid-generated turbulence, *J. Fluid Mech.*, *49*, 611–624, 1971.

Linden, P. F., The formation and destruction of finestructure by double-diffusive processes, *Deep-Sea Res.*, *23*, 895–908, 1976.

Linden, P. F., and J. E. Weber, The formation of layers in a double-diffusive system with a sloping boundary, *J. Fluid Mech.*, *81*, 757–773, 1977.

Mann, C. C., The end of Moore's law?, *MIT Enterprise Technology Review*, *May issue*, online at www.technologyreview.com/magazine/may00/mann.asp. 2000.

Marmorino, G. O., Substructure of oceanic salt finger interfaces, *J. Geophys. Res.*, *94*, 4891–4904, 1989.

Marmorino, G. O., Intrusions and diffusive interfaces in a salt finger staircase, *Deep-Sea Res.*, *38*, 1431–1454, 1991.

May, B. D., Double-diffusive interleaving in baroclinic ocean fronts, Ph.D. thesis, Dalhousie Univ., Halifax, NS, Canada, 1999.

May, B. D., and D. E. Kelley, Effect of baroclinicity on double-diffusive interleaving, *J. Phys. Oceanogr.*, *27*, 1997–2008, 1997.

May, B. D., and D. E. Kelley, Growth and steady state stages of thermohaline intrusions in the Arctic Ocean, *J. Geophys. Res.,106*, 16,783-16,, 2001.

McDougall, T. J., and J. R. Taylor, Flux measurements across a finger interface at low values of the stability ratio, *J. Mar. Res.*, *42*, 1–14, 1984.

Merryfield, W. J., Origin of thermohaline staircases, *J. Phys. Oceanogr.*, *30*, 1046–1068, 2000.

Merryfield, W. J., 2001, Intrusions in double-diffusively stable Arctic waters: evidence for differential mixing?, *J. Phys. Oceanogr.*, *submitted*.

Merryfield, W. J., and M. Grinder, 2001, Salt fingering fluxes from numerical simulations, *J. Phys. Oceanogr.*, *submitted*.

Merryfield, W. J., G. Holloway, and A. E. Gargett, Differential vertical transport of heat and salt by weak stratified turbulence, *Geophys. Res. Lett.*, *25*, 2773–2776, 1998.

Merryfield, W. J., G. Holloway, and A. E. Gargett, A global ocean model with double-diffusive mixing, *J. Phys. Oceanogr.*, *29*, 1124–1142, 1999.

Molemaker, M. J., and H. Dijkstra, The formation and evolution of a diffusive interface, *J. Fluid Mech.*, *331*, 199–229, 1997.

Niino, H., A linear stability theory of double-diffusive horizontal intrusions in a temperature-salinity front, *J. Fluid Mech.*, *171*, 71–1100, 1986.

Özgökmen, T. M., O. Esenkov, and D. Olson, 1998, A numerical study of layer formation due to fingers in double-diffusive convection in a vertically-bounded domain, *J. Mar. Res.*, *56*, 463–487, 1998.

Padman, L., Momentum fluxes through sheared oceanic thermohaline steps, *J. Geophys. Res.*, *99*, 22, 491–22, 499, 1994.

Padman, L., and T. Dillon, Vertical heat fluxes through the beaufort sea thermohaline staircase, *J. Geophys. Res.*, *92*, 10, 799–10, 806, 1987.

Padman, L., and T. M. Dillon, On the horizontal extent of the Canada Basin thermohaline steps, *J. Phys. Oceanogr.*, *18*, 1458–1462, 1988.

Padman, L., and T. M. Dillon, Thermal microstructure and internal waves in the canada basin diffusive staircase, *Deep-Sea Res.*, *36*, 531–542, 1989.

Perkin, R. G., and E. L. Lewis, Mixing in the West Spitsbergen current, *J. Phys. Oceanogr.*, *14*, 1315–1325, 1984.

Phillips, O. M., Turbulence in a strongly stratified fluid – is it unstable?, *Deep-Sea Res.*, *19*, 79–81, 1972.

Raymond, E. S., and B. Young, *The Cathedral and the Bazaar: Musings on Linux and Open Source by an Accidental Revolutionary*, O'Reilly and Associates Press, Sebastopol, CA, USA, 2001.

Ruddick, B., Differential fluxes of heat and salt: implications for circulation and ecosystem modelling, *Oceanography*, *10(3)*, 122–127, 1997.

Ruddick, B. R., and J. S. Turner, The vertical length scale of double-diffusive intursions, *Deep-Sea Res.*, *26A*, 903–913, 1979.

Ruddick, B. R., T. J. McDougall, and J. S. Turner, The formation of layers in a uniformly stirred density gradient, *Deep-Sea Res.*, *36*, 597–609, 1989.

Ruddick, B. R., O. Phillips, and J. Turner, A laboratory and quantitative model of finite-amplitude intrusions, *Dyn. Atmos. Oceans*, *30*, 71–99, 1998.

Rudels, B., G. Björk, R. D. Muench, and U. Schauer, Double-diffusive layering in the Eurasian Basin of the Arctic Ocean, *J. Mar. Sys.*, *21*, 3–27, 1999.

Schmitt, R. W., Flux measurements on salt fingers at an interface, *J. Mar. Res.*, *37*, 419–436, 1979.

Schmitt, R. W., Double diffusion in oceanography, *Ann. Rev. Fluid Mech.*, *26*, 255–285, 1994.

Schmitt, R. W., The salt finger experiments of Jevons (1857) and Rayleigh (1880), *J. Phys. Oceanogr.*, *25*, 8–17, 1995.

St.Laurent, L., and R. W. Schmitt, The contribution of salt fingers to vertical mixing in the north atlantic tracer release experiment, *J. Phys. Oceanogr.*, *29*, 1404–1424, 1999.

Stern, M. E., The "salt-fountain" and thermohaline convection, *Tellus*, *2*, 172–175. 1960.

Stern, M. E., Lateral mixing of water masses, *Deep-Sea Res.*, *14*, 747–753, 1967.

Stern, M. E., Collective instability of salt fingers, *J. Fluid Mech.*, *35*, 209–218, 1969.

Stern, M. E., and J. S. Turner, Salt fingers and convecting layers, *Deep-Sea Res.*, *16*, 497–511, 1969.

Stommel, H., and K. N. Fedorov, Small scale structure in temperature and salinity near Timor and Mindanao, *Tellus*, *14*, 306–325, 1967.

Stommel, H., A. B. Arons, and D. Blanchard, An oceanographical curiosity: the perpetual salt fountain, *Deep-Sea Res.*, *3*, 152–153, 1956.

Stommel, H. M., The sea of the beholder (excerpt of unpublished autobiography), in *Collected works of Henry M. Stommel*, edited by N. G. Hogg and R. X. Huang, vol. 1, American Meteorol. Soc., Boston, MA, USA, 1995.

Tait, R. I., and M. R. Howe, Some observations of thermohaline stratification in the deep ocean, *Deep-Sea Res.*, *15*, 275–280, 1968.

Toole, J. M., and D. T. Georgi, On the dynamics and effects of double-diffusively driven intrusions, *Prog. Oceanogr.*, *10*, 123–145, 1981.

Turner, J. S., The coupled turbulent transports of salt and heat across a sharp density interface, *Int. J. Heat Mass Transfer*, *8*, 759–767, 1965.

Turner, J. S., Salt fingers across a density interface, *Deep-Sea Res.*, *14*, 599–611. 1967.

Turner, J. S., a, The behaviour of a stable salinity gradient heated from below, *J. Fluid Mech.*, *33*, 183–200, 1968.

Turner, J. S., b, The influence of molecular diffusivity on turbulent entrainment across a density interface, *J. Fluid Mech.*, *33*, 639–656, 1968.

Turner, J. S., *Buoyancy Effects in Fluids*, Cambridge University Press, Cambridge, UK, 1973.

Wells, M. G., R. W. Griffiths, and J. S. Turner, The generation of density fine-structure by salt fingers in a spatially periodic shear, *J. Geophys. Res.*, *in press*, 2001.

Zhang, J., and R. W. Schmitt, The impact of salt fingering on the thermohaline circulation under mixed boundary conditions, *J. Phys. Oceanogr.*, *30*, 1223–1231, 2000.

Zhang, J., R. W. Schmitt, and R. X. Huang, Sensitivity of the GFDL modular ocean model to parameterization of double-diffusive processes, *J. Phys. Oceanogr.*, *28*, 589–605, 1998.

Dan E. Kelley, Dalhousie University, Halifax, NS, Canada (email: Dan.Kelley@Dal.Ca)

This preprint was prepared with AGU's LaTeX macros v4, with the extension package 'AGU^{++}' by P. W. Daly, version 1.6a from 1999/05/21, with modifications by D. E. Kelley, version 1.0 from 2001/03/26, for the *'Aha Huliko'a* Hawaiian Winter Workshop.